Decay of semiclassical massless Dirac fermions from integrable and chaotic cavities

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Apr. 26 2019

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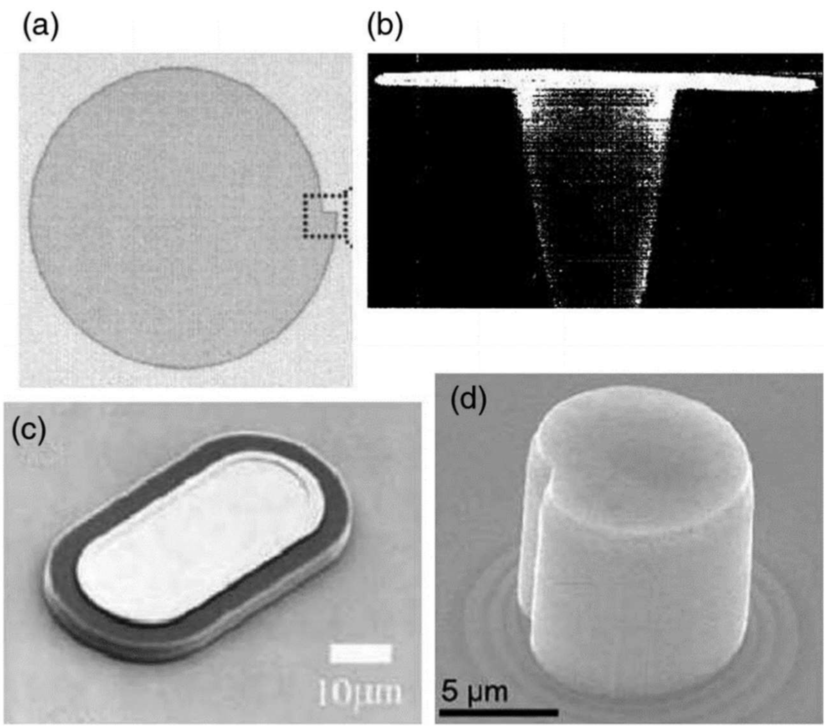
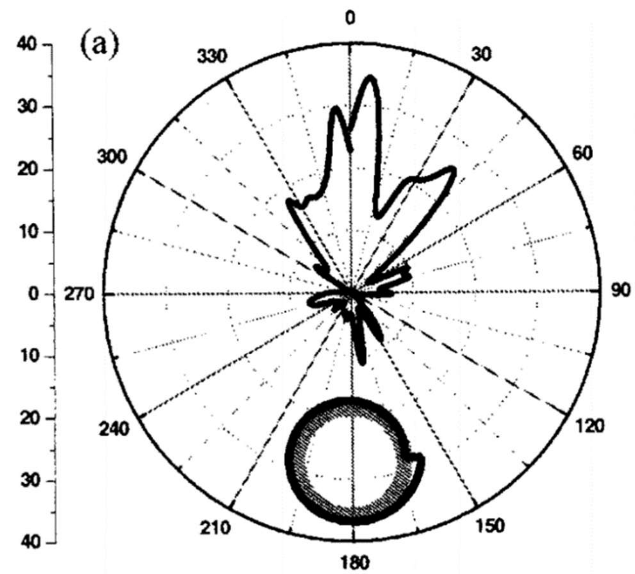
Outline

- Background of dielectric confinement problems in graphene
- Semiclassical regime and ray dynamics
- Decay in integrable and chaotic cavities
- Comparing decaying behavior of Dirac fermion (Pseudospin ½) and Light

- Light cavity



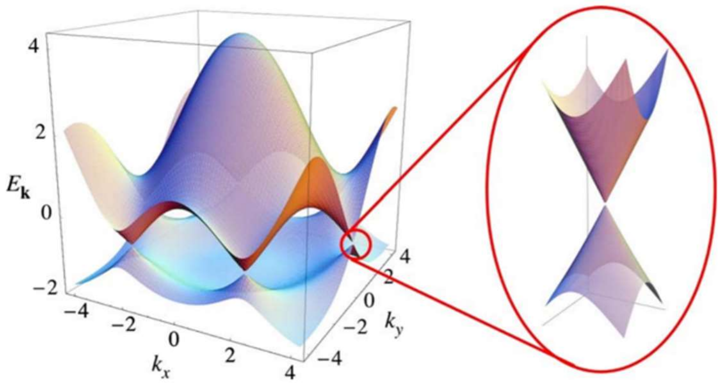


Cao, Hui, and Jan Wiersig. *Rev. Mod. Phys*. **87**.61 (2015)

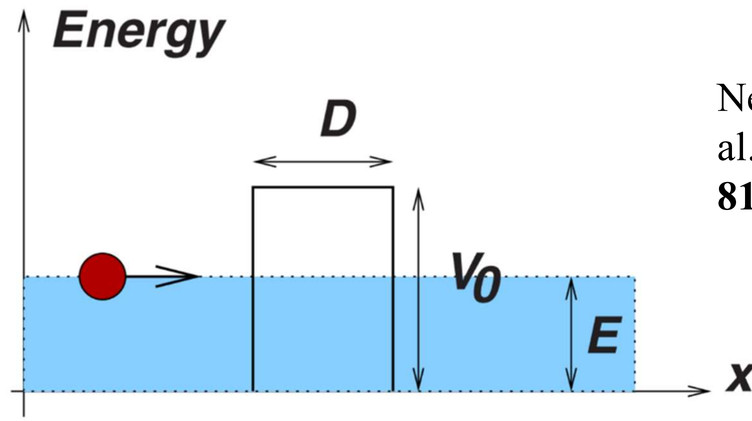
Ben-Messaoud, T., and J. Zyss, 2005, *Appl. Phys. Lett*. **86**, 241110.

Background: The electronic properties of graphene

- Linear dispersion relation

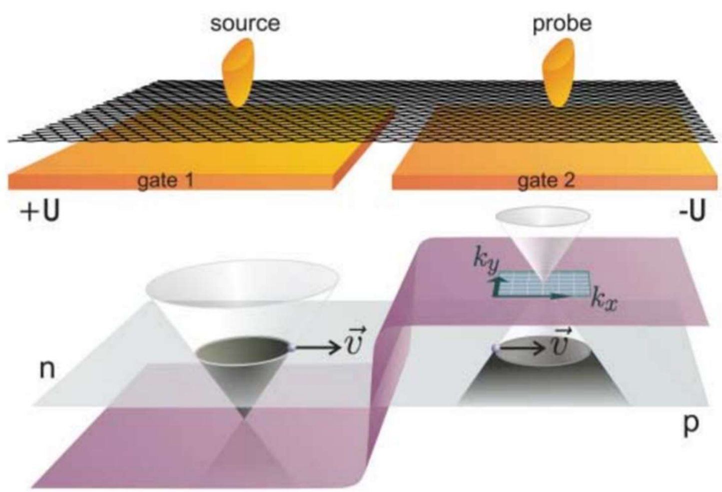


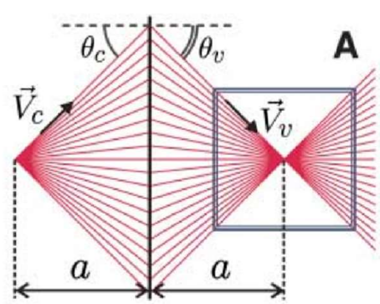
- Klein tunneling



Neto, AH Castro, et al. *Rev. Mod. Phys*. **81**, 109 (2009)

- Veselago Lens

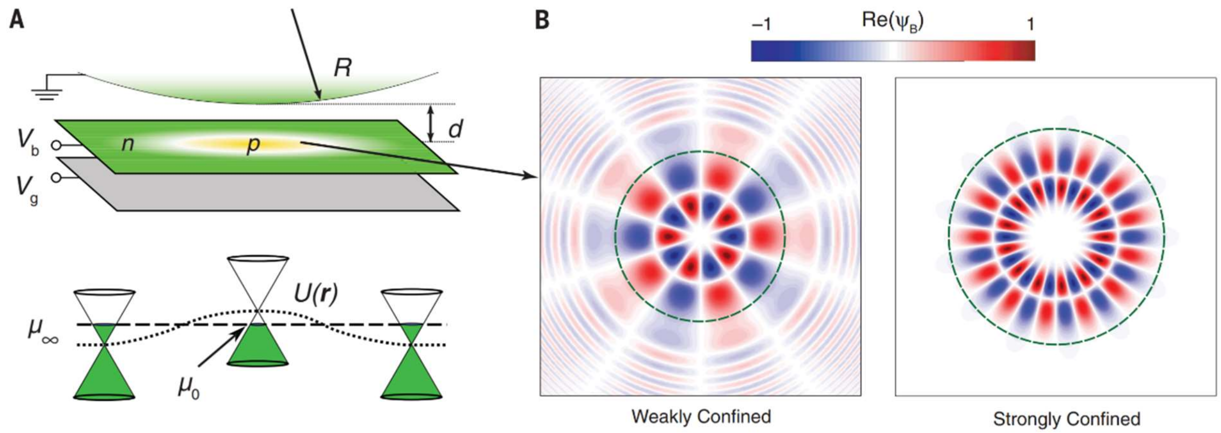




Cheianov, Vadim V., Vladimir Fal'ko, and B. L. Altshuler. *Science* **315**, 5816 (2007)

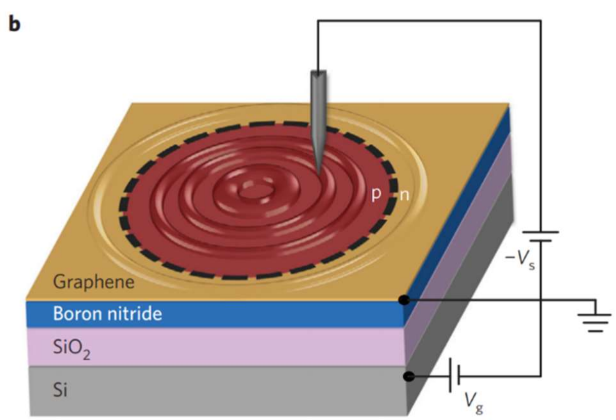
Background: Electrostatically confined Dirac fermions

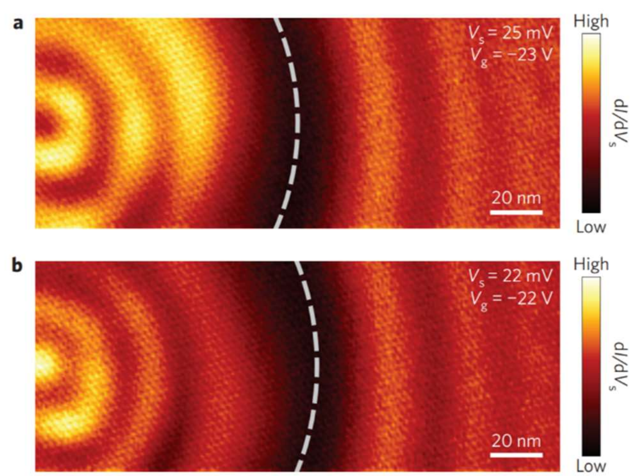
- Cavity: STM-tip or Dopping



Zhao, Yue, et al. *Science* **348**, 672 (2015)

Ghahari, Fereshte, et al. *Science* **356**, 845 (2017).





Lee, Juwon, et al. *Nat. Phys*. **12**, 1032 (2016)

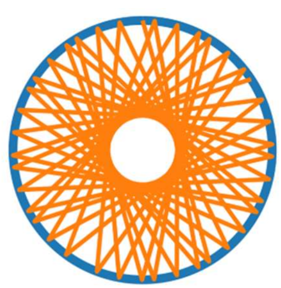
Velasco Jr, Jairo, et al. *Nano Lett*. **16**, 1620 (2016)

Semiclassical regime

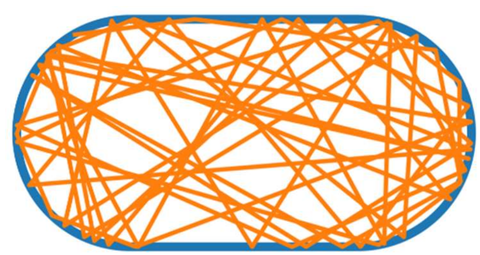
- Is good confinement possible in graphene?

Ryu, Jung-Wan, et al. *Phys. Rev. E*, **73**, 036207 (2006)

Trapping in a cavity → Ray dynamics

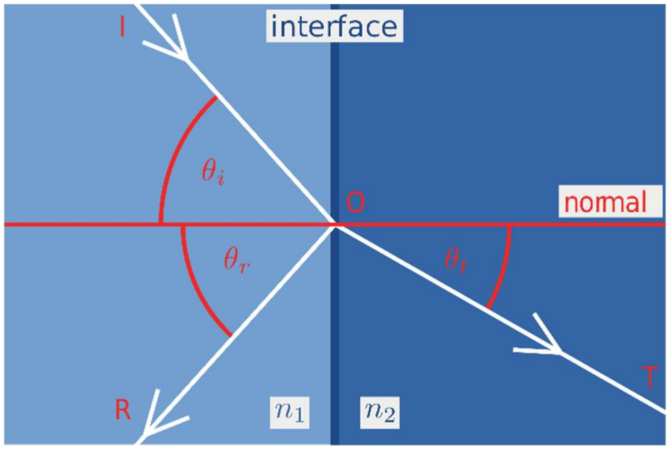


Integrable cavity



Chaotic cavity

➢ Fresnel equations

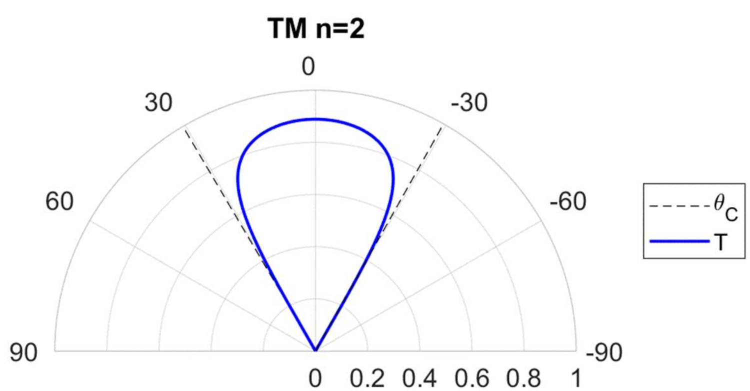


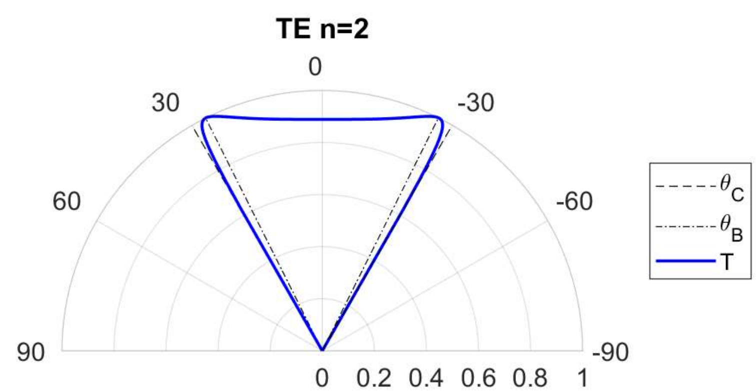
$$n = n_1/n_2$$

$$R_{\mathrm{TM}}(\theta) = \left(\frac{n\ \cos\ \theta - \cos\ \theta_t}{n\ \cos\ \theta + \cos\ \theta_t}\right)^2,$$

$$R_{\mathrm{TE}}(\theta) = \left(\frac{\cos\ \theta - n\ \cos\ \theta_t}{\cos\ \theta + n\ \cos\ \theta_t}\right)^2.$$

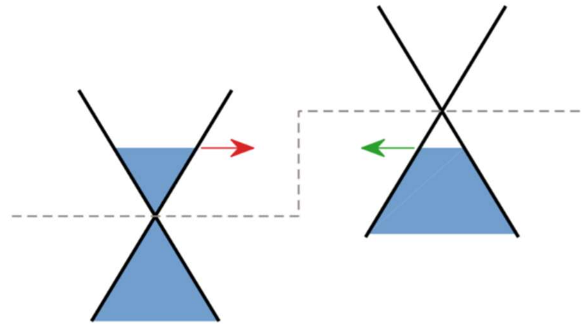
$$T = 1 - R$$

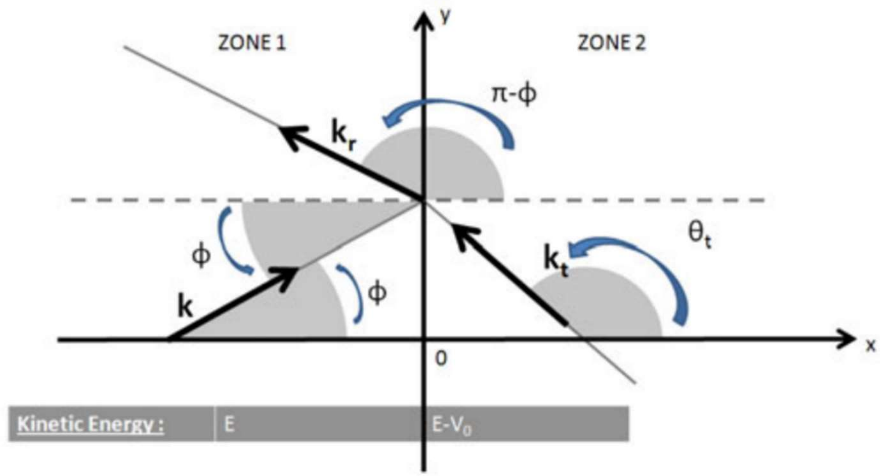




- Klein tunneling

$V_0 > E$





P. E. Allain and J.-N. Fuchs. *Eur. Phys. J. B* **83**, 301 (2011).

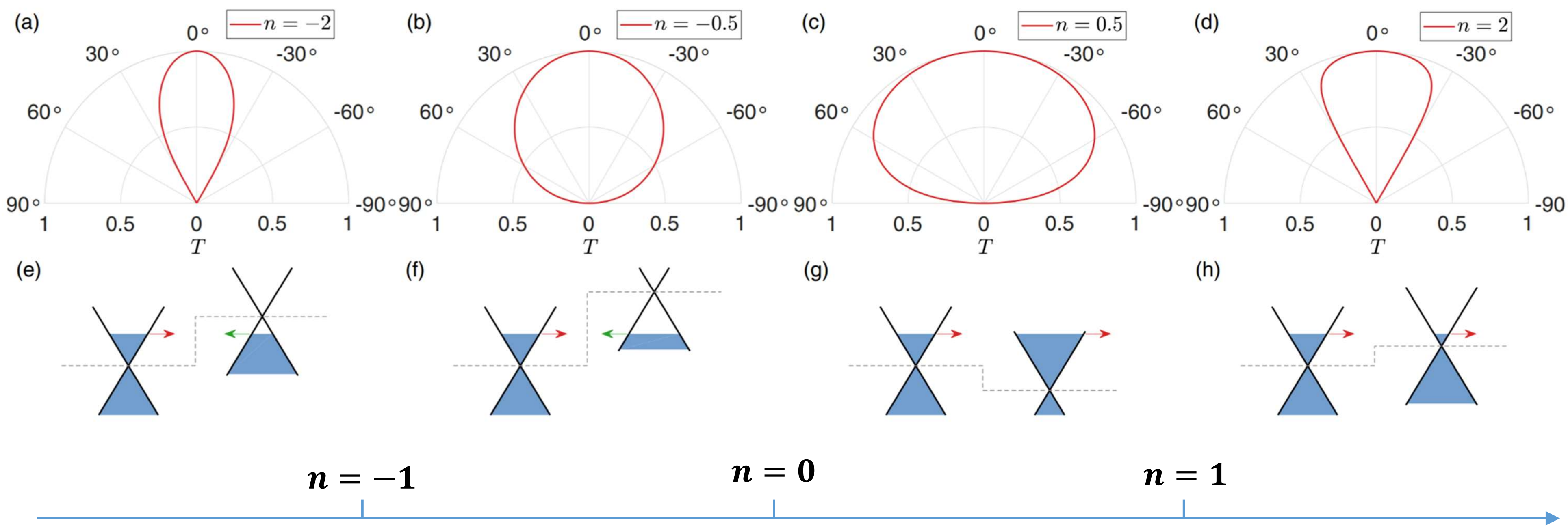
$$\theta_t = \sin^{-1}\left(\frac{E}{E - V_0}\sin\theta\right) + \pi = \sin^{-1}(n\sin\theta) + \pi,$$

$$n = \frac{E}{E - V_0}$$

Klein Tunneling

➢ Reflection and transmission

P. E. Allain and J.-N. Fuchs. *Eur. Phys. J. B* **83**, 301 (2011).



$$T = -\frac{2\cos\theta\cos\theta_t}{1-\cos(\theta+\theta_t)},$$

$$R = \frac{1+\cos(\theta-\theta_t)}{1-\cos(\theta+\theta_t)},$$

$$\theta_t = \sin^{-1}\left(\frac{E}{E-V_0}\sin\theta\right)+\pi = \sin^{-1}(n\sin\theta)+\pi,$$

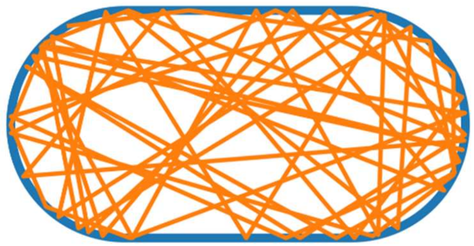
$$T = \frac{2\cos\theta\cos\theta_t}{1+\cos(\theta+\theta_t)},$$

$$R = \frac{1-\cos(\theta-\theta_t)}{1+\cos(\theta+\theta_t)},$$

$$\theta_t = \sin^{-1}(n\sin\theta),$$

Survival probability

$$I_n = I_0 \prod_i R_i = I_0 \exp\left(\sum_i \ln\, R_i\right).$$



Exponential decay $$P(t) = \exp(-\gamma t),$$

Algebraic decay $$P(t) \sim t^{-z},$$

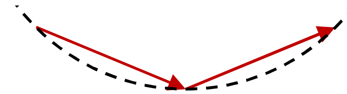
Numerical Calculation

$10^5 \sim 10^7$ Rays with random initial conditions

Decay in circular cavity (n<1)

- $|n| < 1$

No critical angle

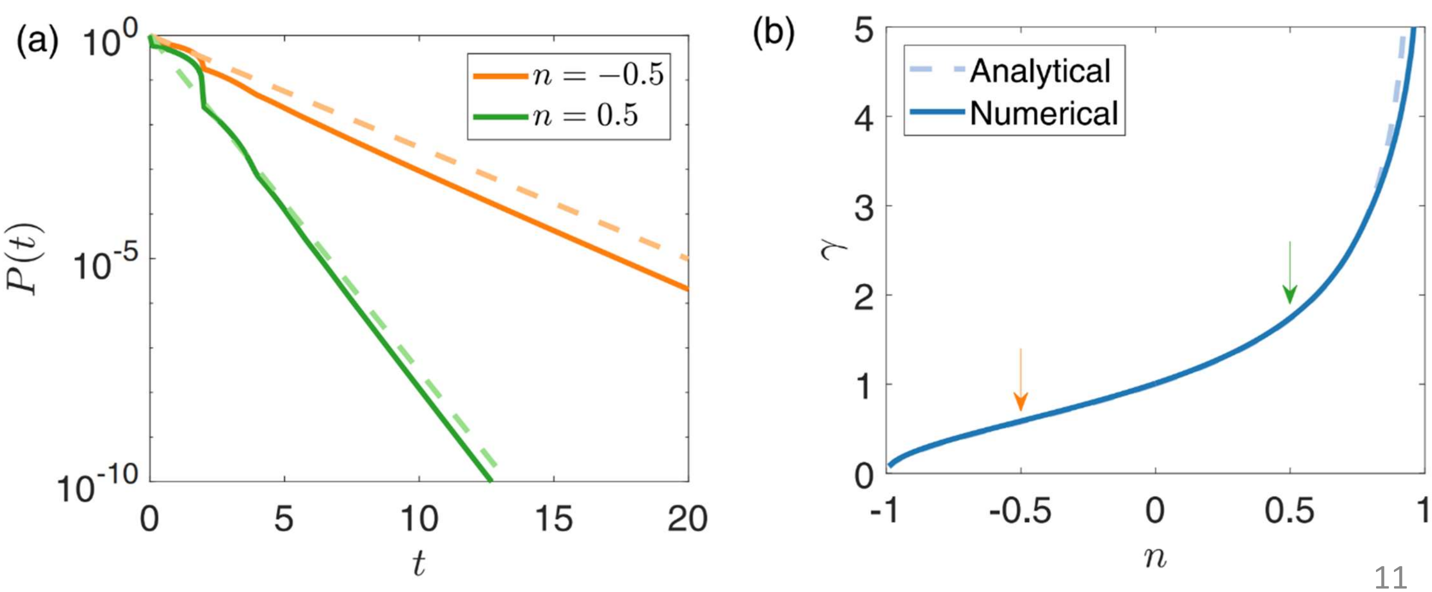


Theory

$$P(t) = \frac{\sum_i R_i^{t/\Delta t}}{N} = \lim_{\theta \to \pi/2} \left(\frac{1 - \cos(\theta - \theta_t)}{1 + \cos(\theta + \theta_t)} \right)^{t/(2\cos\theta)}.$$

$$P(t) \approx \exp\left(-\frac{\sqrt{1-n^2}}{1-n} t \right).$$

Numerical



Decay in circular cavity (n>1)

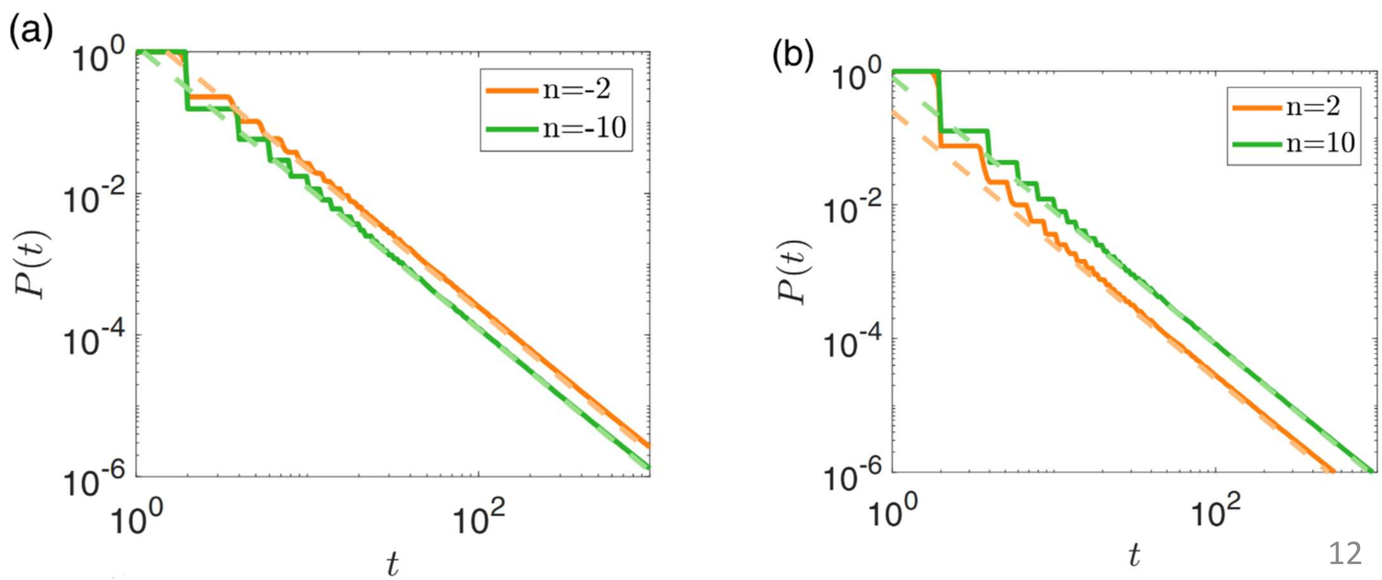
- $|n| > 1$ **With critical angle**

Theory

$$P(t) \approx \frac{2\sqrt{n^2-1}}{\alpha^2 t^2}[1-(1+\alpha\sqrt{\theta_c}t)e^{-\alpha\sqrt{\theta_c}t}]$$
$$\approx \frac{2\sqrt{n^2-1}}{\alpha^2}t^{-2}.$$

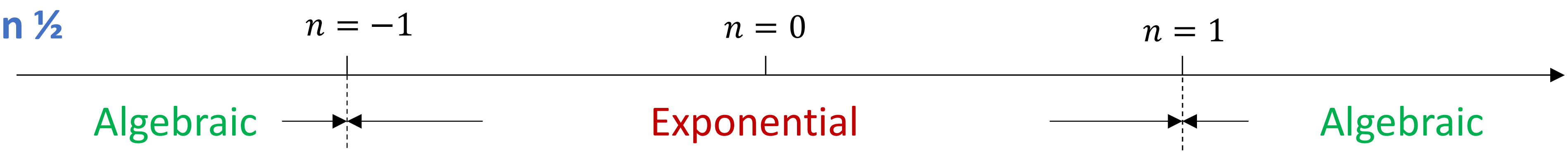
$$\alpha = \frac{n\sqrt{2\sqrt{n^2-1}}}{n-1}.$$

Numerical



Summary for circular cavity

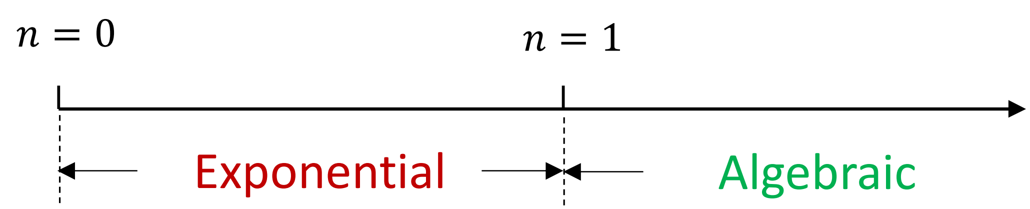
Spin ½



- $n = 1$ $\quad V_0 = 0$ $\quad$ No cavity
- $n = -1$ $\quad P(t) \approx \int_0^{\pi/2} x \exp\left(-\frac{tx}{2}\right) d\theta = \frac{4}{t^2},$

Ryu, Jung-Wan, et al. *Phys. Rev. E*, **73**, 036207 (2006)

Light

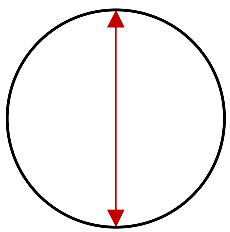


Slowest decaying orbit

- What's the difference between light and pesudospin ½ particle?

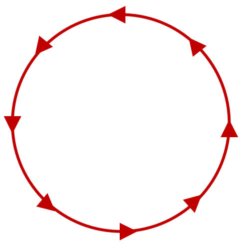
$|n| < 1$ **No critical angle**

Searching for slowest decaying orbit



- Light (TM, TE): $\theta = 0$

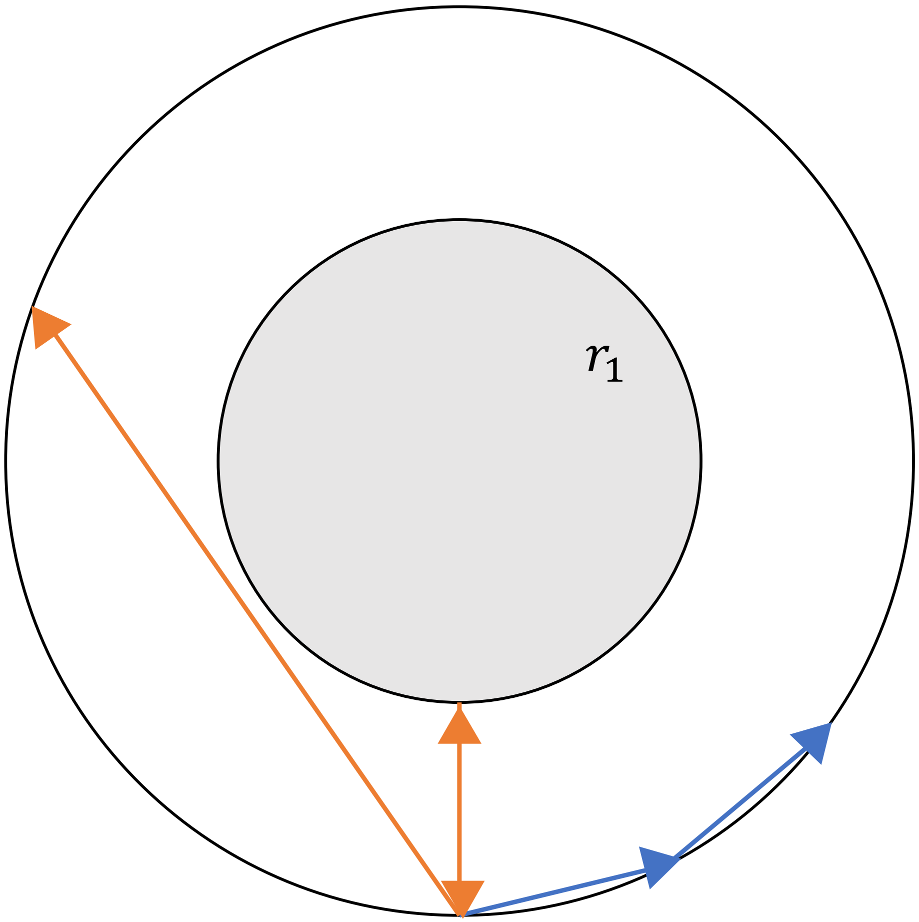
Along the diameter



- Psuedospin ½ : $\theta = \pi/2$

Along the boundary

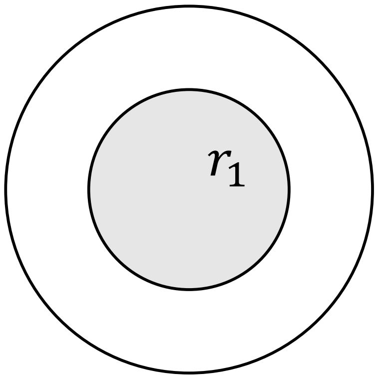
Block



Light: Slowest decaying orbit gets blocked by the center

Spin ½ : Orbit along the boundary is unchanged

Decay in ring cavity



Block

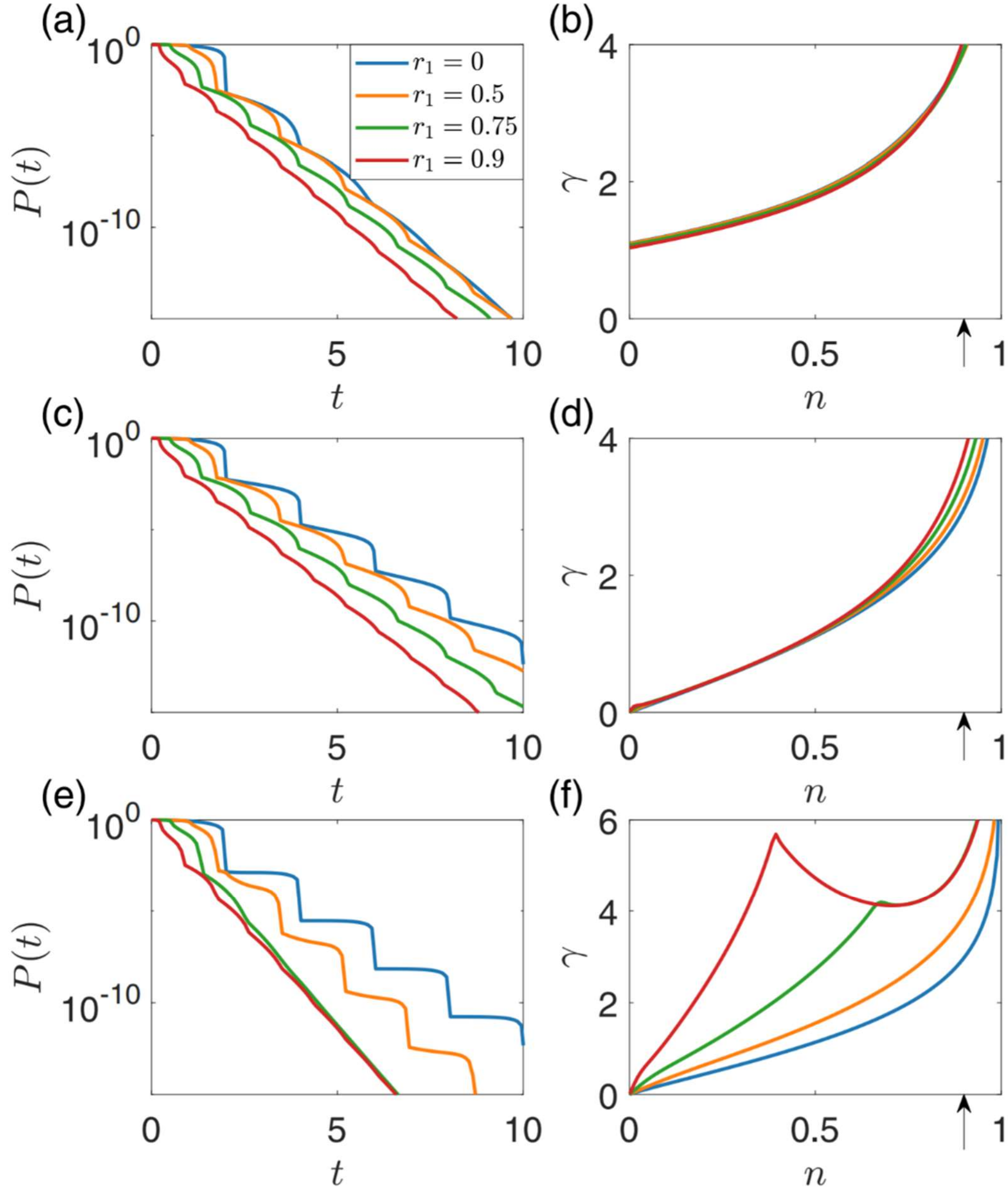
$$n(r) = \begin{cases} \infty, & r \leqslant r_1, \\ (0, 1), & r_1 < r \leqslant 1, \\ 1, & r > 1, \end{cases}$$

$$P(t) = \exp(-\gamma t),$$

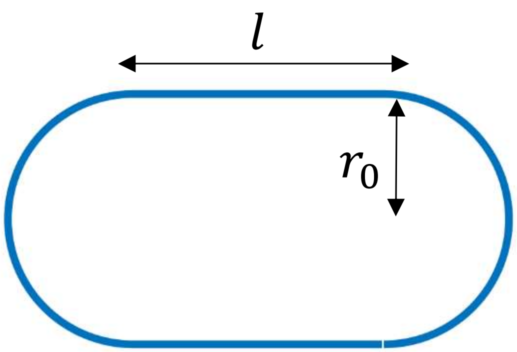
Spin ½

TM

TE



Stadium (chaotic) cavity - theory



Exponential $$P(t) = \exp(-\gamma t),$$

$n \to 0$ $$\gamma = \frac{1}{\langle d \rangle}\left(\sum_{i=1}^{\infty} \frac{W_{2i+1}}{i} + \frac{\pi}{2} n\right),$$

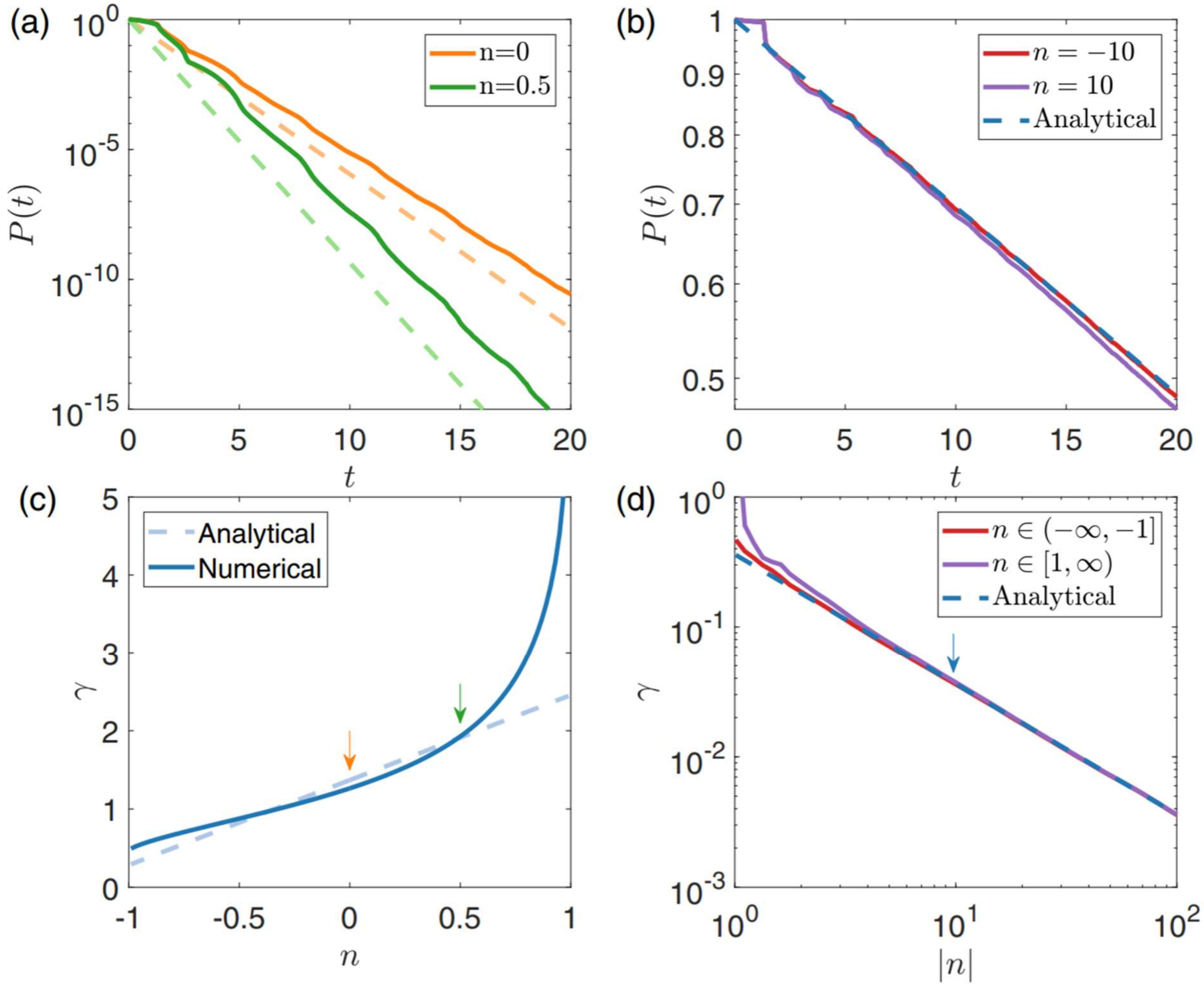
$n \to \infty$ $$\gamma = \frac{(8 - 2\pi)\pi r_0}{\langle d \rangle |n| L}.$$

$$W_n = \int_0^{\pi/2} \cos^n x \, dx = \frac{\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\Gamma\left(\frac{n}{2}+1\right)},$$

Perimeter $L = 2\pi r_0 + 2l,$

Average spacing $\langle d \rangle = \pi A / L$

Stadium (chaotic) cavity - numerical



Mean escape time

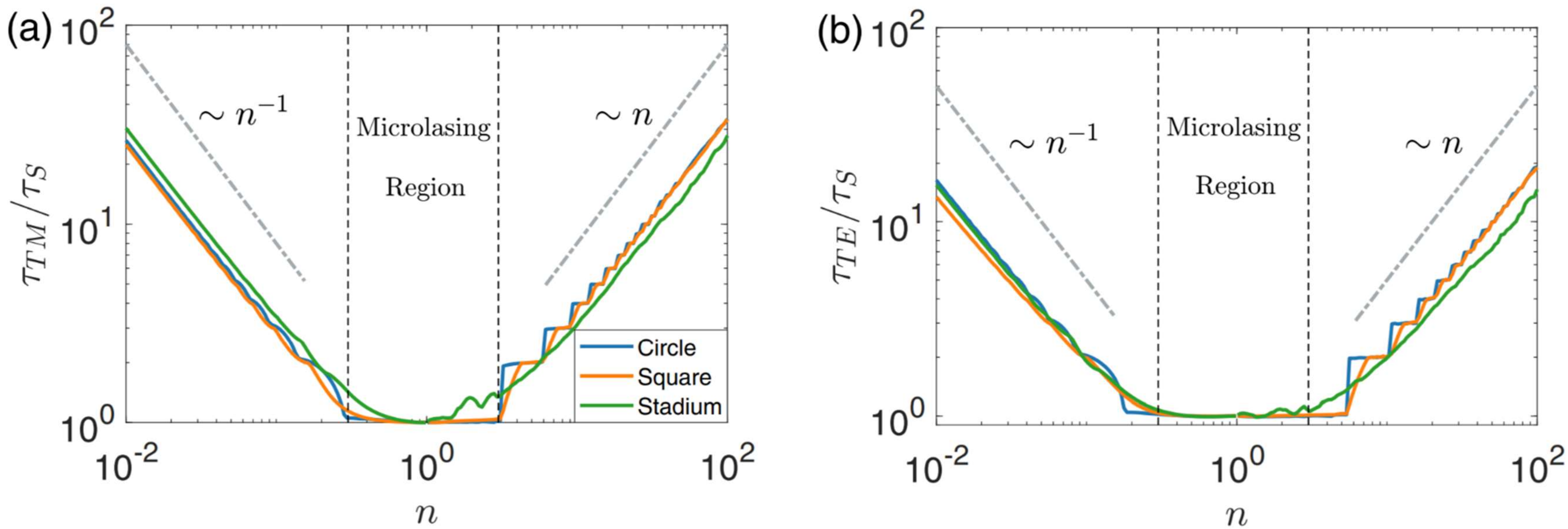
	Circular cavity		Stadium cavity	
	$\lvert n\rvert < 1$	$\lvert n\rvert > 1$	$\lvert n\rvert < 1$	$\lvert n\rvert > 1$
Spin 1/2	$\gamma = \frac{\sqrt{1-n^2}}{1-n}$	$P(t) = \frac{(n-1)^2}{n^2} t^{-2}$	$\gamma = \sum_i \frac{W_{2i+1}}{\langle d\rangle i} + \frac{\pi n}{2\langle d\rangle}$	$\gamma = \frac{(8-2\pi)\pi r_0}{\langle d\rangle \lvert n\rvert L}$
TM	$\gamma = \ln\left(\frac{1+n}{1-n}\right)$	$P(t) = \frac{(n^2-1)^2}{4n^2} t^{-2}$	$\gamma = \frac{n\pi}{\langle d\rangle}$	$\gamma = \frac{2\pi^2 r_0}{\langle d\rangle L n^2}$
TE	$\gamma = \ln\left(\frac{1+n}{1-n}\right)$	$\gamma = \ln\left(\frac{n+1}{n-1}\right)$	$\gamma = \frac{2n\pi}{\langle d\rangle}$	$\gamma = \frac{4\pi^2 r_0}{\langle d\rangle L n^2}$

Define: mean escape time (or lifetime) τ $\qquad P(\tau) = 1/e$

$$\tau_{\mathrm{EM}}/\tau_{\mathrm{S}} = \alpha_1 n^{-1} \quad \text{for } n \ll 1,$$

$$\tau_{\mathrm{EM}}/\tau_{\mathrm{S}} = \alpha_2 n \quad \text{for } n \gg 1,$$

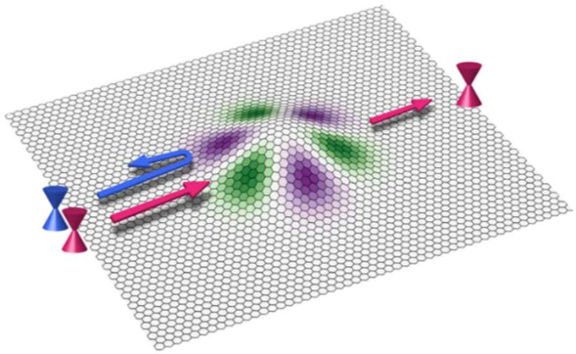
- Stable regime, regardless of chaos



Laser: stimulated emission
Dirac Electron Optics: coherent emission

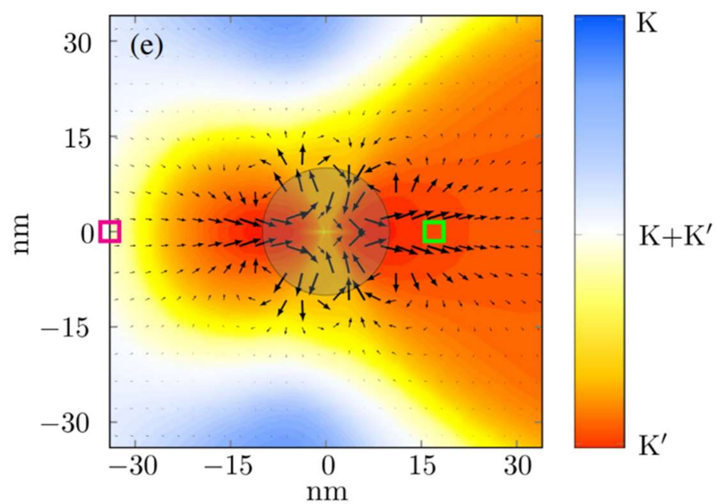
- Valley filters

Controllable strain fields generated by STM tip

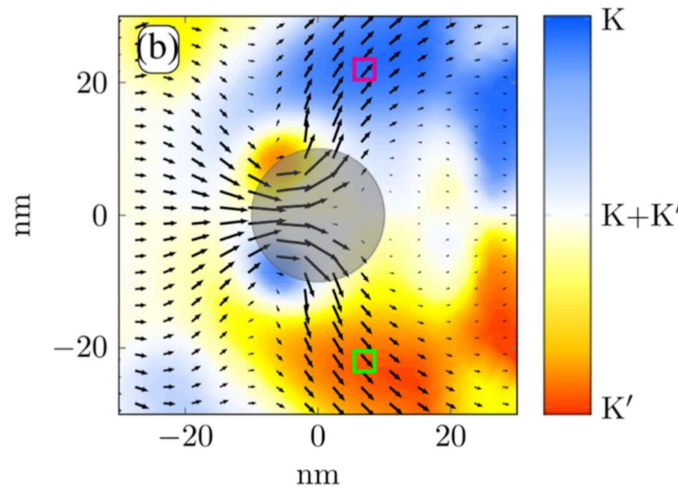


Klimov, Nikolai N. et al. *Science* **336**. 6088 (2012)

Settnes, Mikkel, et al. *Phys. Rev. Lett*. **117**. 276801 (2016)



Out of plane strain



In plane strain

Summary

Previous results

- Survival probability for TM, TE wave
- Good confinement is found in n>1, circular cavity

Our results

- Survival probability for pseudospin ½ system
- Good confinement is found in |n|>1, circular cavity
- Difference between light and pseudospin ½ system
- Confining Dirac fermion is possible for some regime regardless of classical chaos

Publications

- **Han, Chen-Di,** Cheng-Zhen Wang, Hong-Ya Xu, Danhong Huang, and Ying-Cheng Lai. "Decay of semiclassical massless Dirac fermions from integrable and chaotic cavities." *Physical Review B* **98**, 104308 (2018)

- Wang, Cheng-Zhen, **Chen-Di Han**, Hong-Ya Xu, and Ying-Cheng Lai. "Chaos-based Berry phase detector." *Physical Review B* **99**, no. 14 (2019): 144302.

- **Han, Chen-Di,** Hong-Ya Xu, Danhong Huang, and Ying-Cheng Lai. "Atomic collapse in pseudospin-1 systems." *revised PRB*

- **Han, Chen-Di,** Hong-Ya Xu, and Ying-Cheng Lai. “Super confinement for pseudospin X systems." *in prepared*

- **Han, Chen-Di,** Hong-Ya Xu, and Ying-Cheng Lai. “Out-of-time-order correlator in relativistic quantum integrable and chaotic systems." *to be resubmitted*

ARIZONA STATE UNIVERSITY

Thanks for your attention !