Out-of-Time-order Correlator in billiard systems

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CH1

Outline

1. Background
2. Numerical result
   1) Short time behavior
   2) Long time behavior
3. Analytical analysis
4. Conclusion

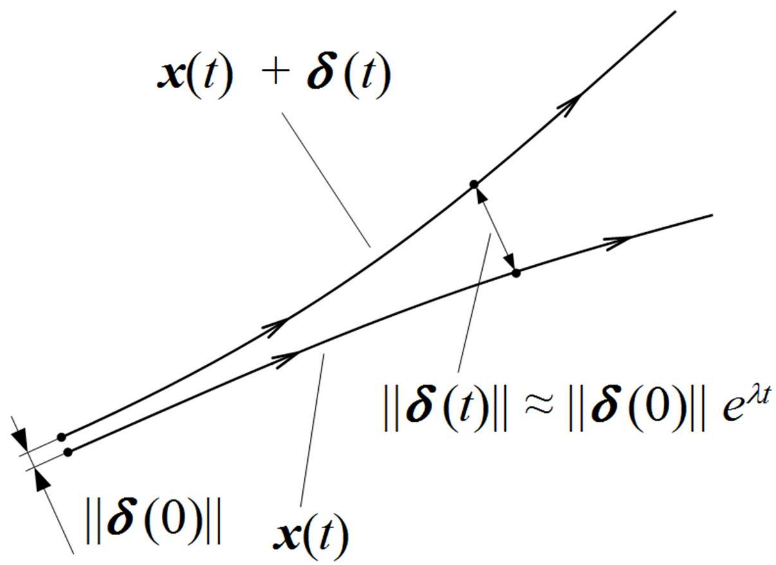
C.-D. Han, H.-Y. Xu and Y.-C. Lai, "Out-of-Time-order Correlator in billiard systems," submitted to *Physical Review Letters*.

Slide 2

CH1 Chendi Han, 3/3/2018

Classical chaos

Lyapunov exponent characterizes the rate of separation of infinitesimally close trajectories



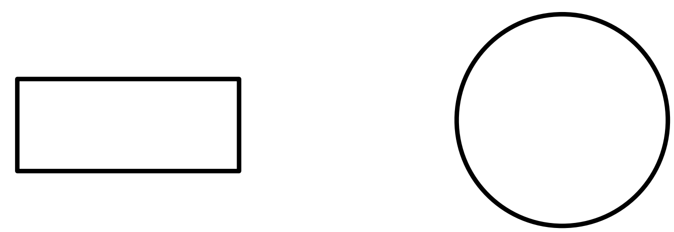
From: Wiki

Quantum chaos

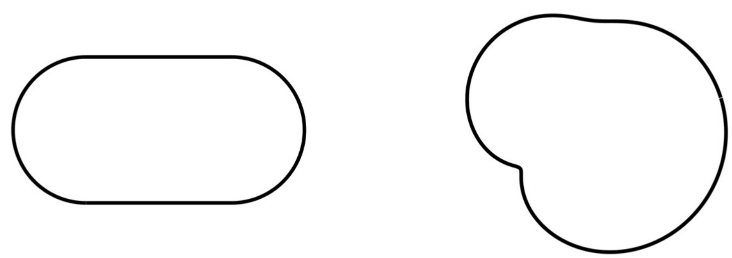
Classical chaotic system $\longrightarrow$ **Quantum chaos**

Billiard system:

Integrable

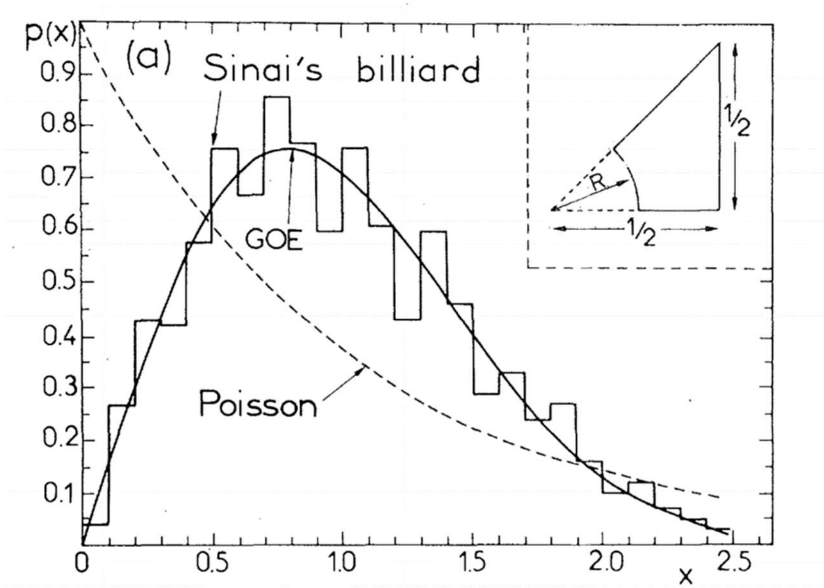


Chaotic



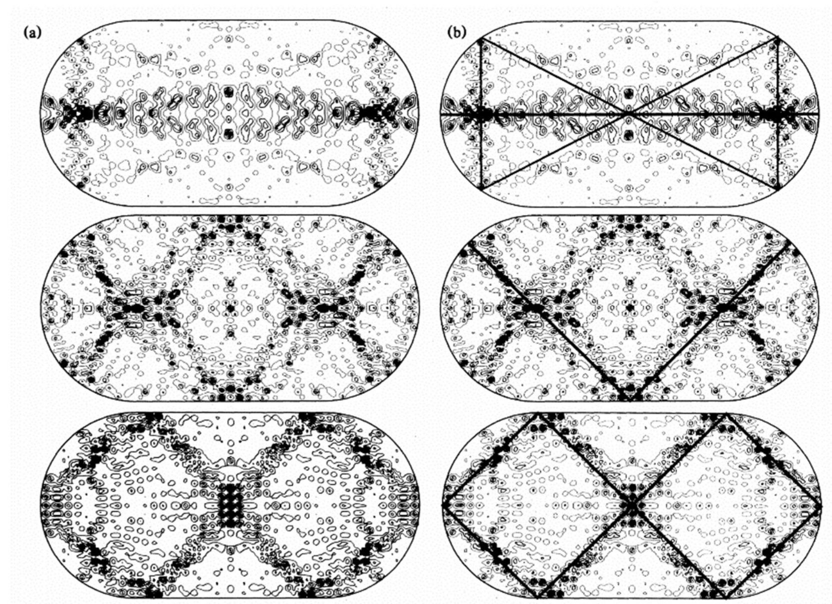
Quantum chaos

BGS conjecture: Random matrix theory



Bohigas, Oriol, Marie-Joya Giannoni, and Charles Schmit. "Characterization of chaotic quantum spectra and universality of level fluctuation laws." Physical Review Letters 52.1 (1984): 1.

Quantum scar



Heller, Eric J. "Bound-state eigenfunctions of classically chaotic Hamiltonian systems: scars of periodic orbits." Physical Review Letters 53.16 (1984): 1515.

Quantum-Classical correspondence

	Nonlinear dynamics (**Classical mechanics**)		**Quantum mechanics**
Two Order	$\frac{\Delta x(t)}{\Delta x(0)} \sim e^{\gamma t}$	$\{x(t), p(0)\}_P$	$\frac{[\hat{x}(t), \hat{p}(0)]}{i\hbar}$
Four Order		$\langle \{x(t), p(0)\}_P^2 \rangle$	$-\langle [\hat{x}(t), \hat{p}(0)]^2 \rangle_\beta$ $\quad \beta = 1/T$

Out-of-Time-order Correlator

$$C_T(t) = -\langle [\hat{W}(t), \hat{V}(0)]^2 \rangle_\beta$$

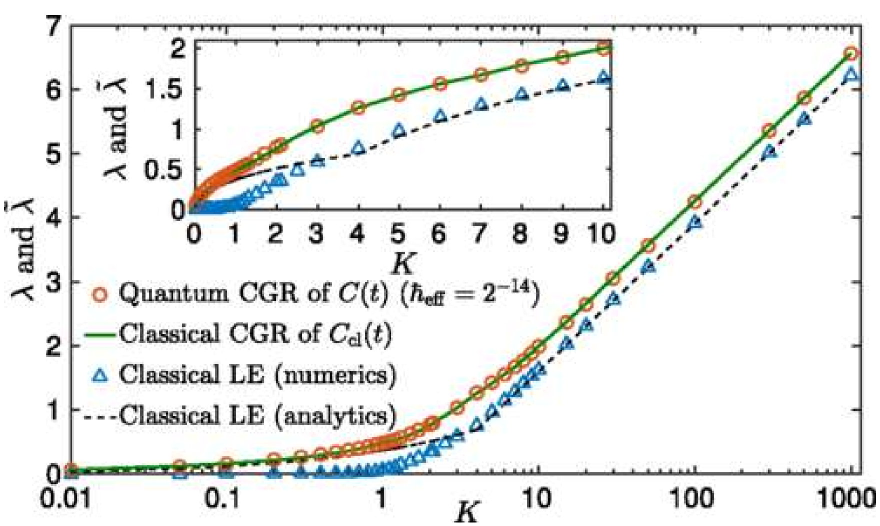
Larkin, A. I., and Yu N. Ovchinnikov. "Quasiclassical method in the theory of superconductivity." Sov Phys JETP 28.6 (1969): 1200-1205.
Maldacena, Juan, Stephen H. Shenker, and Douglas Stanford. "A bound on chaos." *Journal of High Energy Physics* 2016.8 (2016): 106.

$$C_T(t) = \frac{\sum_n c_n \exp\left(-|E_n|/T\right)}{\exp\left(-|E_n|/T\right)}$$

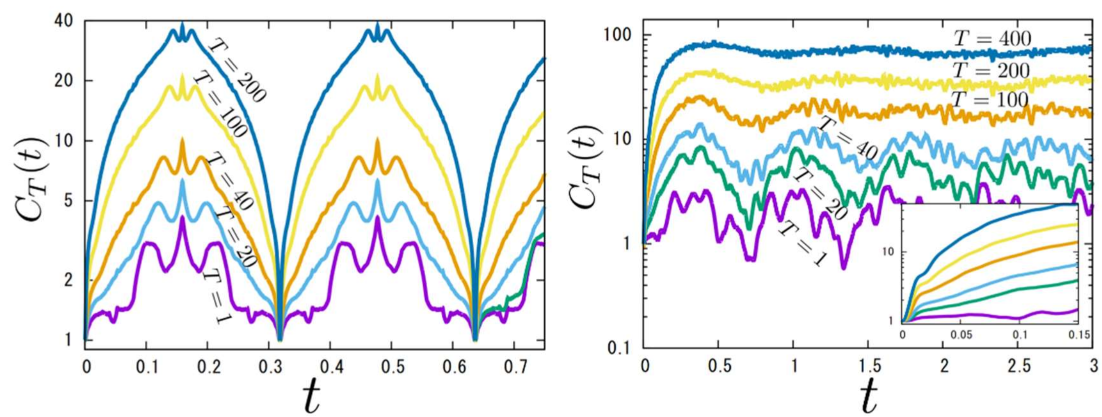
$$c_n(t) = -\langle n | [\hat{W(t)}, \hat{V}]^2 | n \rangle$$

Quantum-Classical discordance

Kicked Rotor



Schrodinger Billiard



(a) Particle in a box

(b) Stadium billiard

Rozenbaum, Efim B., Sriram Ganeshan, and Victor Galitski. "Lyapunov exponent and out-of-time-ordered correlator's growth rate in a chaotic system." Physical review letters 118.8 (2017): 086801.

Hashimoto, Koji, Keiju Murata, and Ryosuke Yoshii. "Out-of-time-order correlators in quantum mechanics." Journal of High Energy Physics 2017.10 (2017): 138.

Model

Schrodinger system

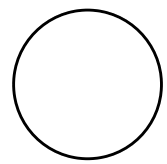
$$\hat{H} = \frac{\hat{\mathbf{p}}}{2m} + V(\mathbf{r})$$

Dirac system

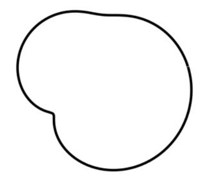
$$\hat{H} = \hat{\boldsymbol{\sigma}} \cdot \hat{\mathbf{p}} + V(\mathbf{r})\hat{\sigma}_z$$

$$V(\mathbf{r}) = \begin{cases} 0 & \mathbf{r} \in D, \\ \infty & \mathbf{r} \in D \end{cases}$$

Circle billiard



African billiard



$$C_T(t) = -\langle [\hat{W}(t), \hat{V}(0)]^2 \rangle_\beta$$

$$\boxed{\hat{W} = \hat{V} = \frac{\partial \hat{H}}{\partial \hat{p}}}$$

$$C_T(t) = -\langle [\hat{p}_x(t), \hat{p}_x]^2 \rangle_\beta \quad \sim \left(\frac{\Delta p_x(t)}{\Delta x(0)} \right)^2$$

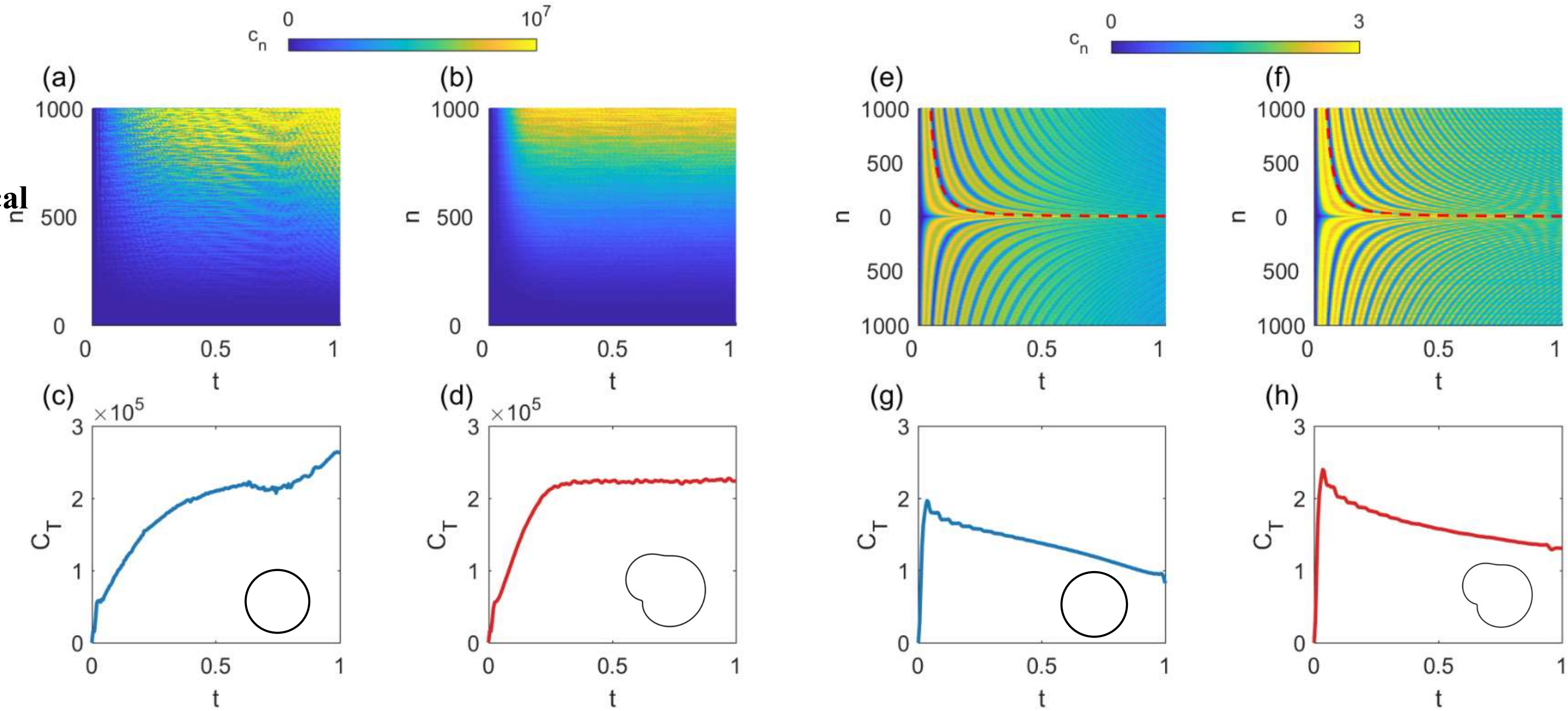
$$C_T(t) = -\langle [\hat{\sigma}_x(t), \hat{\sigma}_x]^2 \rangle_\beta$$

Classical view: Wave-packet

Result: Small time $t < t_R$

Micro-canonical OTOC

Canonical OTOC



$$t = \frac{2\pi}{2|E_n|}$$

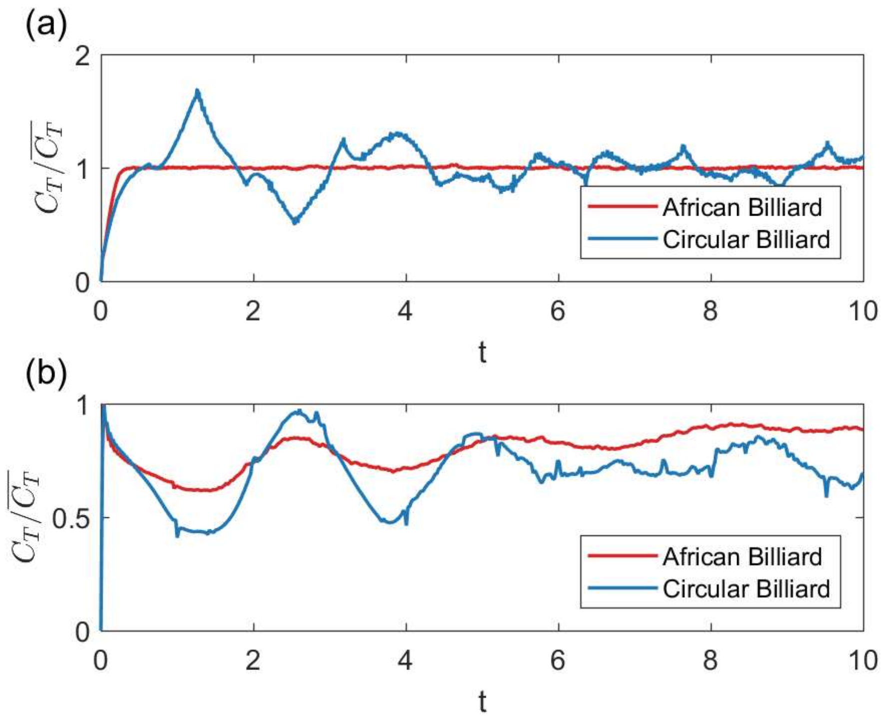
Schrodinger system $(a)-(d)$

Dirac system $(e)-(h)$

Result: Long time $t > t_R$

Schrodinger system

Dirac system



Long time behavior could diagnolized quantum chaos

$$\overline{C}_T(t) = \langle \hat{W}(t)\hat{V}\hat{V}\hat{W}(t) + \hat{V}\hat{W}(t)\hat{W}(t)\hat{V}\rangle_\beta$$

Analytical analysis

$$C_T(t) = -\langle [\hat{W}(t), \hat{V}(0)]^2 \rangle_\beta$$

1. Find the maximum value
2. Truncate and make approximation

1D system: Harmonic oscillator

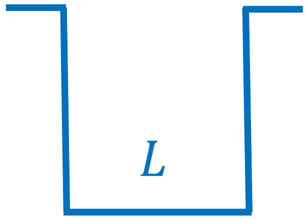
$$C_T = -\langle [\hat{x}(t), \hat{p}_x]^2 \rangle_\beta = \left\langle \left(\frac{\Delta x(t)}{\Delta x(0)} \right)^2 \right\rangle = \boldsymbol{(\cos t)^2}$$

$$C_T = -\langle [\hat{p}(t), \hat{p}_x]^2 \rangle_\beta = \left\langle \left(\frac{\Delta p_x(t)}{\Delta x(0)} \right)^2 \right\rangle = \boldsymbol{(\omega \sin t)^2}$$

Hashimoto, Koji, Keiju Murata, and Kentaroh Yoshida. "Chaos in chiral condensates in gauge theories." *Physical review letters* 117.23 (2016): 231602.

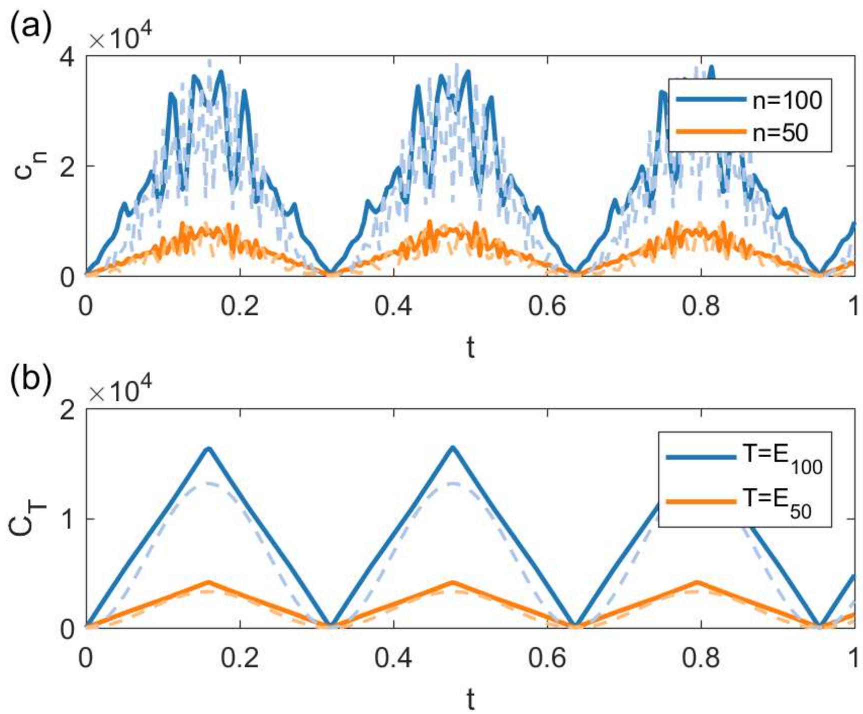
Analytical analysis

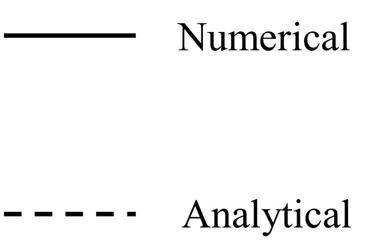
1D system: Infinite potential well



$$c_n(t) = \frac{64 n^2 L^2}{\pi^4}\left[4\sin^4\left(\frac{\pi^2}{L^2}t\right)\sin^2\left(\frac{\pi^2 n}{L^2}t\right) + 1 - \cos\left(\frac{2\pi^2}{L^2}t\right)\right]$$

$$C_T(t) = \frac{64 L^4}{\pi^6} T\left[1 - \cos\left(\frac{2\pi^2}{L^2}t\right)\right]$$

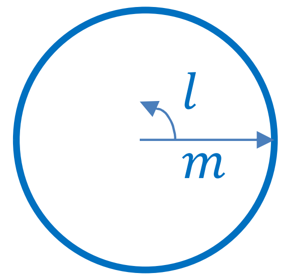




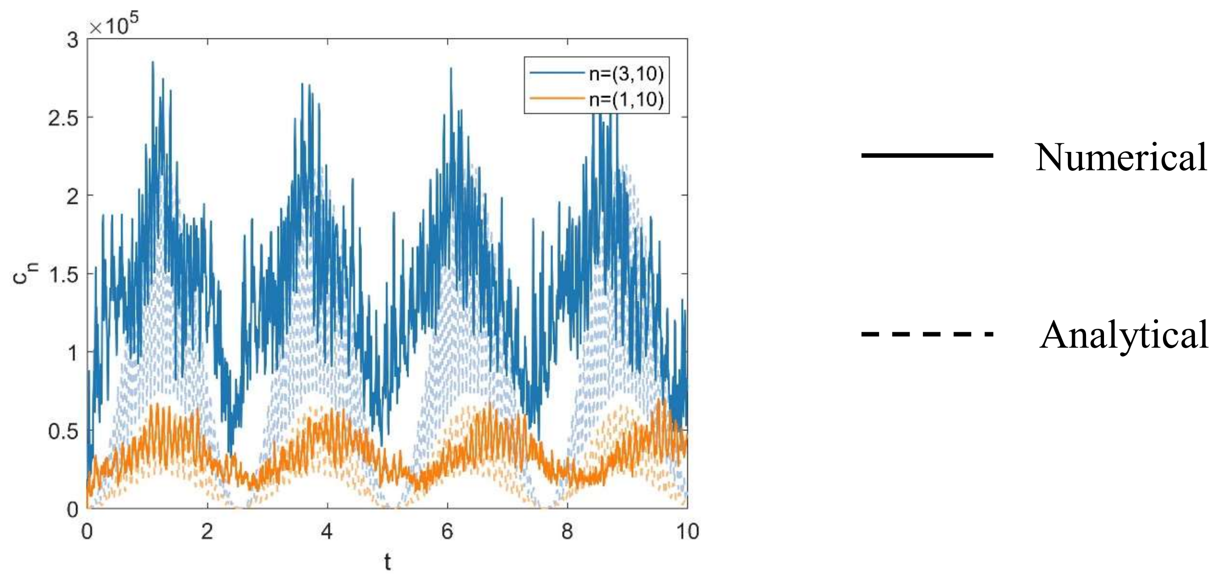
Analytical analysis

2D system:

Circular billiard
(Schrodinger system)

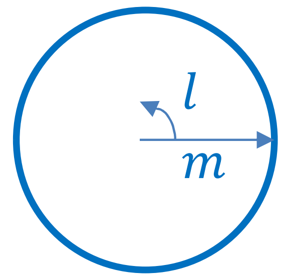


$$c_{ml} \approx 16x_m^4 k_{ml}^4 d_m^4 \left[\sin^2\left(\frac{d_m^2}{\hbar}\right)\cos^2\left(\frac{2k_{ml}d_m}{\hbar}\right)\right] + 4x_m^4 k_{ml}^4 d_m^4 \left(1 - \cos\frac{2d_m^2}{\hbar}\right) \qquad l \gg m$$



Analytical analysis

2D system: Circular billiard (Dirac system)



$$C_T(t) = -\langle[\hat{W}(t), \hat{V}(0)]^2\rangle_\beta$$

Particle-Particle: **Canceled**

Particle-Hole: **Particle-Hole symmetry**

$$\hat{A} = \hat{U}\hat{K}$$

$$\hat{H}' = \hat{A}\hat{H}\hat{A}^{-1} = -\hat{H}$$

Zitterbewegung

Mondragon, R. J. "Neutrino billiards: time-reversal symmetry-breaking without magnetic fields." *Proc. R. Soc. Lond. A*. Vol. 412. No. 1842. The Royal Society, 1987.
Bjorken, James D., and Sidney David Drell. "Relativistic quantum mechanics." (1964).

Analytical analysis

Wave-packet revival: periodic motion for wave-packet

$$E(n) \approx E(n_0) + E'(n_0)(n-n_0) + \frac{E''(n_0)}{2}(n-n_0)^2 + \frac{E'''(n_0)}{6}(n-n_0)^3 + \cdots \ ,$$

$$T_{\rm cl} = \frac{2\pi\hbar}{|E'(n_0)|}, \quad T_{\rm rev} = \frac{2\pi\hbar}{|E''(n_0)|/2}, \quad \text{and} \quad T_{\rm super} = \frac{2\pi\hbar}{|E'''(n_0)|/6} \ .$$

Infinite potential well(Schrodinger system) $$T_0 = T_{\rm rev} = \frac{4mL^2}{\pi\hbar}$$

Circular billiard (Schrodinger system) $$T_0 = 4T_{\rm rev} = \frac{8mR^2}{\pi\hbar}$$

Circular billiard (Dirac system) $$T_0 = T_{cl} = 4R$$

Robinett, R. W., and S. Heppelmann. "Quantum wave-packet revivals in circular billiards." *Physical Review A* 65.6 (2002): 062103.
Robinett, Richard W. "Quantum wave packet revivals." *Physics Reports* 392.1-2 (2004): 1-119.

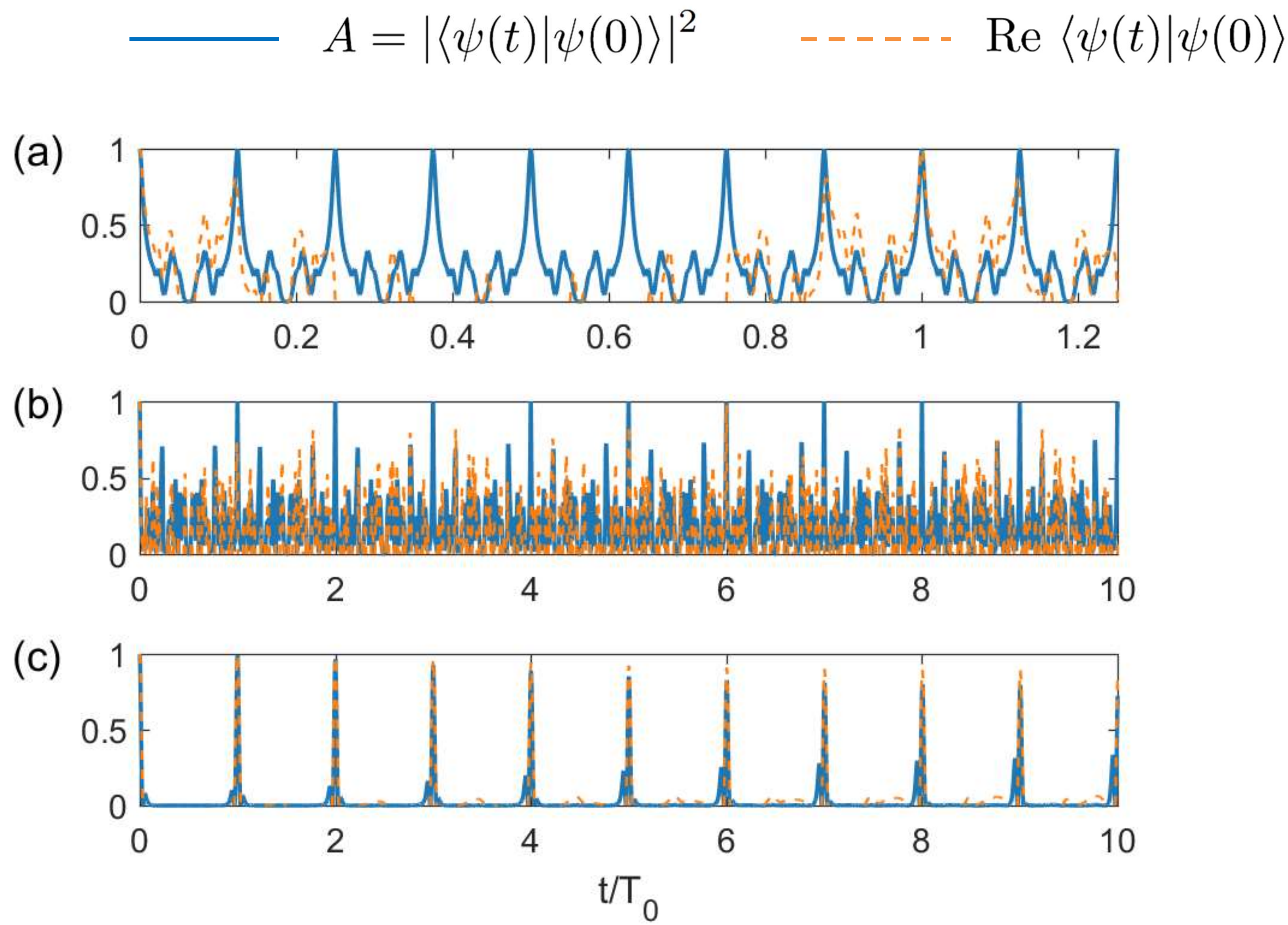
Analytical analysis

Wave-packet revival: periodic motion for wave-packet

Infinite potential well

Circular billiard
(Schrodinger system)

Circular billiard
(Dirac system)

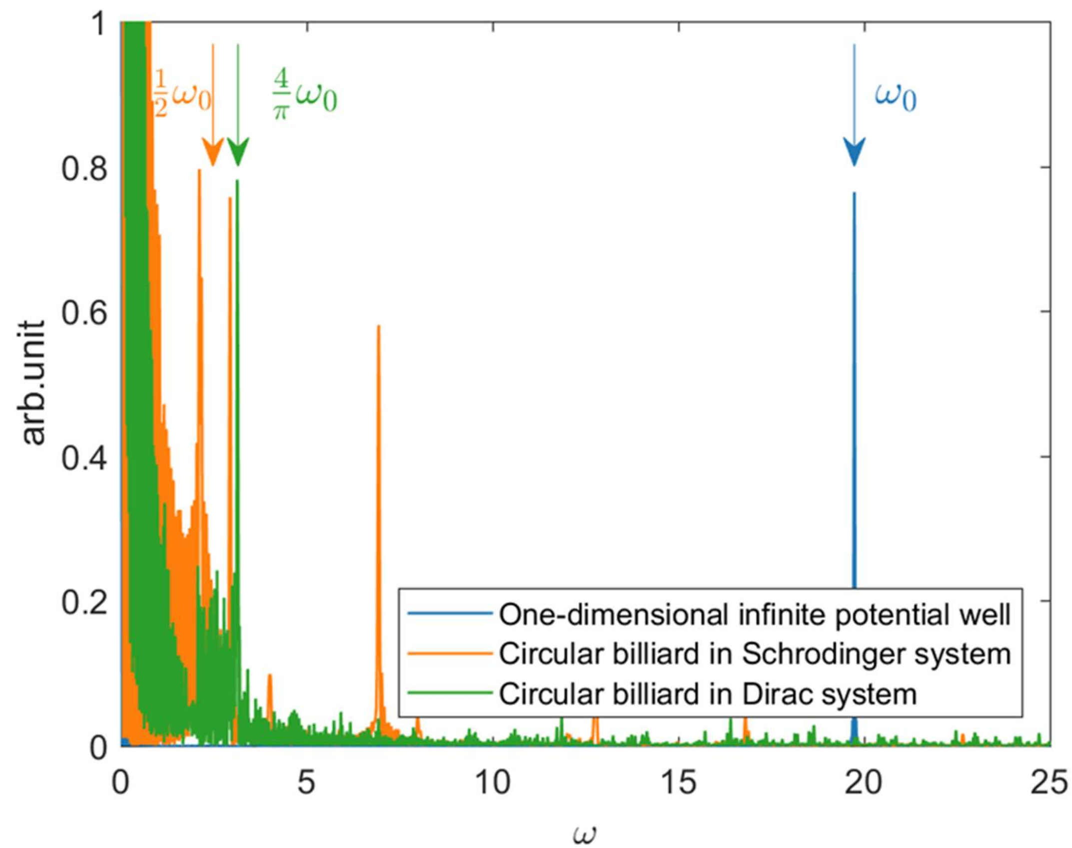


Center wave-packet $2\sigma \sim \frac{1}{10}$ system size $k_0 = 0$

Analytical analysis

Fourier Analysis for $C_T(t)$ $T = E_{100}$

Infinite potential well	T_0
Circular billiard (Schrodinger system)	$2T_0$
Circular billiard (Dirac system)	$\frac{\pi}{4}T_0$



Conclusion

1. Dynamical independent Zitterbewegung motion in Dirac billiard

Particle-Hole symmetry

2. Long time behavior of OTOC can distinguish quantum chaos

Wave-packet revival

Thank you